NAG Toolbox for MATLAB

d02jb

1 Purpose

d02jb solves a regular linear two-point boundary-value problem for a system of ordinary differential equations by Chebyshev-series using collocation and least-squares.

2 Syntax

```
[c, ifail] = d02jb(n, cf, bc, x0, x1, k1, kp)
```

3 Description

d02jb calculates the solution of a regular two-point boundary-value problem for a regular linear nth-order system of first-order ordinary differential equations in Chebyshev-series in the range (x_0, x_1). The differential equation

$$y' = A(x)y + r(x)$$

is defined by the user-supplied real function **cf** and the boundary conditions at the points x_0 and x_1 are defined by the user-supplied (sub)program **bc**.

You specify the degree of the Chebyshev-series required, $\mathbf{k1} - 1$, and the number of collocation points, **kp**. The function sets up a system of linear equations for the Chebyshev coefficients, n equations for each collocation point and one for each boundary condition. The boundary conditions are solved exactly, and the remaining equations are then solved by a least-squares method. The result produced is a set of coefficients for a Chebyshev-series solution for each component of the solution of the system of differential equations on a range normalized to $(-1, 1)$.

e02ak can be used to evaluate the components of the solution at any point on the interval (x_0, x_1) – see Section 9 for an example. e02ah followed by e02ak can be used to evaluate their derivatives.

4 References

Picken S M 1970 Algorithms for the solution of differential equations in Chebyshev-series by the selected points method *Report Math. 94* National Physical Laboratory

5 Parameters

5.1 Compulsory Input Parameters

1: **n – int32 scalar**

n, the order of the system of differential equations.

Constraint: $\mathbf{n} \geq 1$.

2: **cf – string containing name of m-file**

cf defines the system of differential equations (see Section 3). It must return the value of a coefficient function $a_{i,j}(x)$, of A, at a given point x, or of a right-hand side function $r_i(x)$ if $\mathbf{j} = 0$.

Its specification is:

```
[result] = cf(ii, j, x)
```

Input Parameters

1: **ii – int32 scalar**

2: **j – int32 scalar**

Indicate the function to be evaluated, namely $a_{i,j}(x)$ if $1 \leq \mathbf{j} \leq n$, or $r_i(x)$ if $\mathbf{j} = 0$. $1 \leq \mathbf{ii} \leq n$, $0 \leq \mathbf{j} \leq n$.

3: **x – double scalar**

The point at which the function is to be evaluated.

Output Parameters

1: **result – double scalar**

The result of the function.

3: **bc – string containing name of m-file**

bc defines the n boundary conditions, which have the form $y_k(x_0) = s$ or $y_k(x_1) = s$. The boundary conditions may be specified in any order.

Its specification is:

```
[j, rhs] = bc(ii)
```

Input Parameters

1: **ii – int32 scalar**

The index of the boundary condition to be defined.

Output Parameters

1: **j – int32 scalar**

Must be set to $-k$ if the ith boundary condition is $y_k(x_0) = s$, or to $+k$ if it is $y_k(x_1) = s$.

j must not be set to the same value k for two different values of **ii**.

2: **rhs – double scalar**

The value s.

4: **x0 – double scalar**

5: **x1 – double scalar**

The left- and right-hand boundaries, x_0 and x_1, respectively.

Constraint: **x1** $>$ **x0**.

6: **k1 – int32 scalar**

The number of coefficients to be returned in the Chebyshev-series representation of the components of the solution (hence the degree of the polynomial approximation is **k1** $- 1$).

Constraint: **k1** ≥ 2.

7: **kp – int32 scalar**

The number of collocation points to be used.

Constraint: **kp** $\geq$ **k1** $- 1$.

5.2 Optional Input Parameters

None.

5.3 Input Parameters Omitted from the MATLAB Interface

ldc, w, lw, iw, liw

5.4 Output Parameters

1: **c(ldc,n) – double array**

The computed Chebyshev coefficients of the kth component of the solution, y_k; that is, the computed solution is:

$$y_k = {\sum_{i=1}^{\mathbf{k1}}}' \mathbf{c}(i,k) T_{i-1}(x), \qquad 1 \leq k \leq n$$

where $T_i(x)$ is the ith Chebyshev polynomial of the first kind, and $\sum'$ denotes that the first coefficient, $\mathbf{c}(1,k)$, is halved.

2: **ifail – int32 scalar**

0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

ifail $= 1$

On entry, $\mathbf{n} < 1$,
or $\mathbf{x0} \geq \mathbf{x1}$,
or $\mathbf{k1} < 2$,
or $\mathbf{kp} < \mathbf{k1} - 1$,
or $\mathbf{ldc} < \mathbf{k1}$.

ifail $= 2$

On entry, $\mathbf{lw} < 2 \times \mathbf{n} \times (\mathbf{kp} + 1) \times (\mathbf{n} \times \mathbf{k1} + 1) + 7 \times \mathbf{n} \times \mathbf{k1}$,
or $\mathbf{liw} < \mathbf{n} \times (\mathbf{k1} + 2)$ (i.e., insufficient workspace).

ifail $= 3$

Either the boundary conditions are not linearly independent, (that is, in the user-supplied (sub)program **bc** the variable **j** is set to the same value k for two different values of **ii**), or the rank of the matrix of equations for the coefficients is less than the number of unknowns. Increasing **kp** may overcome this latter problem.

ifail $= 4$

The least-squares function f04am has failed to correct the first approximate solution (see f04am).

7 Accuracy

The Chebyshev coefficients are determined by a stable numerical method. The accuracy of the approximate solution may be checked by varying the degree of the polynomials and the number of collocation points (see Section 8).

8 Further Comments

The time taken by d02jb depends on the size and complexity of the differential system, the degree of the polynomial solution and the number of matching points.

The collocation points in the range (x_0, x_1) are chosen to be the extrema of the appropriate shifted Chebyshev polynomial. If $\mathbf{kp} = \mathbf{k1} - 1$, then the least-squares solution reduces to the solution of a system of linear equations and true collocation results.

The accuracy of the solution may be checked by repeating the calculation with different values of **k1** and with **kp** fixed but $\mathbf{kp} \gg \mathbf{k1} - 1$. If the Chebyshev coefficients decrease rapidly for each component (and consistently for various **k1** and **kp**), the size of the last two or three gives an indication of the error. If the Chebyshev coefficients do not decay rapidly, it is likely that the solution cannot be well-represented by Chebyshev-series. Note that the Chebyshev coefficients are calculated for the range $(-1, 1)$.

Linear systems of high-order equations in their original form, singular problems, and, indirectly, nonlinear problems can be solved using d02tg.

9 Example

d02jb_bc.m

```
function [jj, rhs] = bc(ii)
  rhs = 0;
  if (ii > 1)
    jj = int32(-1);
  else
    jj = int32(1);
  end
```

d02jb_cf.m

```
function result = cf(ii, jj, xx)
  if (jj == ii)
    result = 0;
  elseif (ii == 1 && jj == 2)
    result = 1;
  elseif (ii == 2 && jj == 1)
    result = -1;
  elseif (ii == 1)
    result = 0;
  else
    result = 1;
  end
```

```
n = int32(2);
x0 = -1;
x1 = 1;
k1 = int32(4);
kp = int32(10);
[c, ifail] = d02jb(n, 'd02jb_cf', 'd02jb_bc', x0, x1, k1, kp)
```

```
c =
   -0.7798    0.0000
   -0.0000    1.5751
    0.3899    0.0000
    0.0000   -0.0629
ifail =
           0
```